Text, letter

Description automatically generated

Proof:

Let be arbitrary and fixed.

Let be arbitrary and fixed.

Then .

This is divisible by since

Thus

Therefore is reflexive. ⁍

Proof:

Let be arbitrary and fixed.

Let be arbitrary and fixed.

Assume is true.

We can then write , for some .

Then by the multiplication property of equality.

Then

Thus which is a multiply of .

Therefore .

Therefore is reflexive. ⁍

Proof:

Let be arbitrary and fixed.

Let be arbitrary and fixed.

Assume is true.

By definition of modular congruence and for some .

Thus .

is a multiple of .

Thus

Therefore

Thus is transitive. ⁍

Text

Description automatically generated

Proof:

Let be arbitrary and fixed.

Assume is true. By definition of modular congruence and for some

Then

Then

Let then

Then which is a multiple of .

By definition of modular congruence .

Therefore ⁍

Proof:

Let be arbitrary and fixed. Assume is true.

By definition of modular congruence for some

By the multiplication property of equality

thus is a multiple of .

Thus .

Therefore ( ⁍

Proof:

Let be arbitrary and fixed. Assume is true.

Then by definition of modular congruence and for some .

Then and

Thus

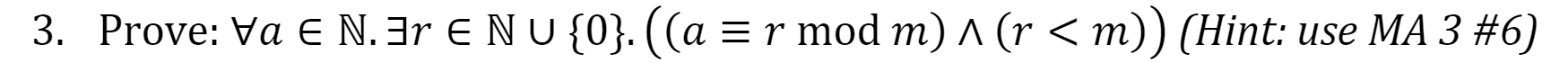
Factoring gives

Let , then

Then

Therefore

Therefore . ⁍



Proof:

Let be arbitrary and fixed. By definition of modular congruence .

Then for some .

We can manipulate the equality to solve for .

Then .

By definition of modular congruence

Then for some .

Then .

Thus, showing the statement is true.

By the Euclidean division theorem for every integer there exists unique integers and such that with .

This shows .

Then , and subsequently shows .

Therefore is a true statement.

Thus proving (⁍

Text

Description automatically generated

Proof:

Let be arbitrary and fixed.

Suppose

Then which is an element of .

Thus holds true and the relation on is reflexive. ⁍

B)

Proof:

Let be arbitrary and fixed.

Assume is true. Then

Case 1:

Then = 1

Thus

Case 2:

Then = 0

Thus

Case 3:

Then

Thus

Therefore (

Therefore, the relation on is symmetric. ⁍

Proof:

Let be arbitrary and fixed.

Assume () is true.

Then and

Let and

Then .

Then

Then which is not element of .

Therefore, the relation on is not transitive. ⁍

Text

Description automatically generated

Indirect: becomes . Assume P is false yet Q is true, find a contradiction.

Case 1:

Assume for the sake of contradiction that elements and are not transitive.

Then

Then .

However, this is a contradiction to the assumption that .

Case 2:

Assume for the sake of contradiction that elements and are transitive .

Then, .

However, this is a contradiction to the assumption that .

This is because elements and are symmetric.

Case 3:

Assume for the sake of contradiction that elements and are transitive.

Then, .

However, this is a contradiction to the assumption that .

This is because elements and are symmetric.

Therefore, the statement is true.

⁍